

7-8 Study Guide and Intervention

Inverse Functions and Relations

Find Inverses

Inverse Relations	Two relations are inverse relations if and only if whenever one relation contains the element (a, b) , the other relation contains the element (b, a) .
Property of Inverse Functions	Suppose f and f^{-1} are inverse functions. Then $f(a) = b$ if and only if $f^{-1}(b) = a$.

Example Find the inverse of the function $f(x) = \frac{2}{5}x - \frac{1}{5}$. Then graph the function and its inverse.

Step 1 Replace $f(x)$ with y in the original equation.

$$f(x) = \frac{2}{5}x - \frac{1}{5} \rightarrow y = \frac{2}{5}x - \frac{1}{5}$$

Step 2 Interchange x and y .

$$x = \frac{2}{5}y - \frac{1}{5}$$

Step 3 Solve for y .

$$x = \frac{2}{5}y - \frac{1}{5}$$

$$5x = 2y - 1$$

$$5x + 1 = 2y$$

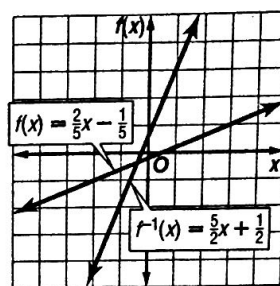
$$\frac{1}{2}(5x + 1) = y$$

Inverse

Multiply each side by 5.

Add 1 to each side.

Divide each side by 2.



The inverse of $f(x) = \frac{2}{5}x - \frac{1}{5}$ is $f^{-1}(x) = \frac{1}{2}(5x + 1)$.

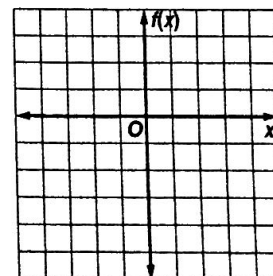
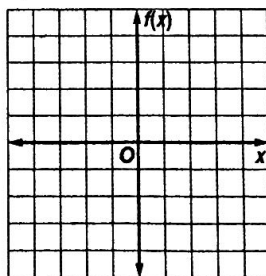
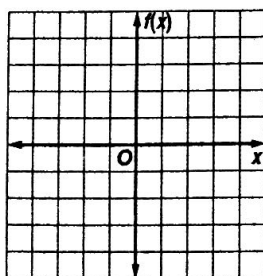
Exercises

Find the inverse of each function. Then graph the function and its inverse.

1. $f(x) = \frac{2}{3}x - 1$

2. $f(x) = 2x - 3$

3. $f(x) = \frac{1}{4}x - 2$



7-8

Study Guide and Intervention *(continued)*

Inverse Functions and Relations

Inverses of Relations and Functions

Inverse Functions

Two functions f and g are inverse functions if and only if $[f \circ g](x) = x$ and $[g \circ f](x) = x$.**Example 1**Determine whether $f(x) = 2x - 7$ and $g(x) = \frac{1}{2}(x + 7)$ are inverse functions.

$$[f \circ g](x) = f[g(x)]$$

$$= f\left[\frac{1}{2}(x + 7)\right]$$

$$= 2\left[\frac{1}{2}(x + 7)\right] - 7$$

$$= x + 7 - 7$$

$$= x$$

$$[g \circ f](x) = g[f(x)]$$

$$= g(2x - 7)$$

$$= \frac{1}{2}(2x - 7 + 7)$$

$$= x$$

The functions are inverses since both $[f \circ g](x) = x$ and $[g \circ f](x) = x$.**Example 2**Determine whether $f(x) = 4x + \frac{1}{3}$ and $g(x) = \frac{1}{4}x - 3$ are inverse functions.

$$[f \circ g](x) = f[g(x)]$$

$$= f\left(\frac{1}{4}x - 3\right)$$

$$= 4\left(\frac{1}{4}x - 3\right) + \frac{1}{3}$$

$$= x - 12 + \frac{1}{3}$$

$$= x - 11\frac{2}{3}$$

Since $[f \circ g](x) \neq x$, the functions are not inverses.**Exercises**

Determine whether each pair of functions are inverse functions.

1. $f(x) = 3x - 1$

$g(x) = \frac{1}{3}x + \frac{1}{3}$

2. $f(x) = \frac{1}{4}x + 5$

$g(x) = 4x - 20$

3. $f(x) = \frac{1}{2}x - 10$

$g(x) = 2x + \frac{1}{10}$

4. $f(x) = 2x + 5$

$g(x) = 5x + 2$

5. $f(x) = 8x - 12$

$g(x) = \frac{1}{8}x + 12$

6. $f(x) = -2x + 3$

$g(x) = -\frac{1}{2}x + \frac{3}{2}$

7. $f(x) = 4x - \frac{1}{2}$

$g(x) = \frac{1}{4}x + \frac{1}{8}$

8. $f(x) = 2x - \frac{3}{5}$

$g(x) = \frac{1}{10}(5x + 3)$

9. $f(x) = 4x + \frac{1}{2}$

$g(x) = \frac{1}{2}x - \frac{3}{2}$

10. $f(x) = 10 - \frac{x}{2}$

$g(x) = 20 - 2x$

11. $f(x) = 4x - \frac{4}{5}$

$g(x) = \frac{x}{4} + \frac{1}{5}$

12. $f(x) = 9 + \frac{3}{2}x$

$g(x) = \frac{2}{3}x - 6$